## Compactifications in Topology

Topology is the study of shapes and spaces. It is fundamental in the study of Mathematics and its influence can be seen across many other areas of study. It is often known as the "mathematics of continuity". Topology like many areas of Mathematics is axiomatic.

**Definition 1**
Let $X$ be a non empty set. $\tau$ is a collection of subsets of $X$ and is defined as a **Topology** if the following axioms are satisfied:

1. $X, \emptyset \in \tau$,
2. The union of **any** (finite or infinite) sets in $\tau$ belongs to $\tau$,
3. The intersection of two sets in $\tau$ belongs to $\tau$

   $\forall x, y \in \tau \quad ; \quad x \cap y \in \tau.$

($X$,$\tau$) is known as a **Topological space**. Where $X$ is a set and $\tau$ is the *topology* of $X$.

Key to understanding topology is the understanding of Topological spaces, and in relation to this, accumulation points and the Bolzano-Weierstrass theorem along with the difference between open and closed sets. Thus enabling us to understand the concepts of closure and density. By clear examples these two concepts allow us to begin to explore compactifications.

**Definition 2**
If $X$ is a topological space and $A \subset X$. The **closure** of $A$, denoted by $\bar{A}$, is the smallest closed set containing $A$.
$A$ is **dense** in $X$ iff $X$ is the smallest closed subspace of $X$. (i.e. $X \Leftrightarrow \bar{A}$=$X$)
If these hold for an $A \subset X$ then $A$ is **compact** or $X$.

Leading from the above we can then discuss, in relation to compactness, compact sets, subsets of compact spaces and the finite intersection property. Once compactness has been outlined we can proceed to Hausdorff compactifications and Local Compactness, also known as *one-point compactification.* We can then explore more types of compact spaces/sets, such as sequentially compact sets, countably compact sets and Stone-Čech properties of Locally compact spaces.

Some examples of these concepts are best shown in metric spaces. This section will outline the axioms and properties of metric topologies and spaces along with the Cauchy sequence with which we are enabled to explain compactness within a metric space.

I aim to clearly define and show examples of compactifications by use of outlining the basic essential theorems of topology and thus forming a solid foundation upon which it should be understandable for those reading this project to understand topology applied to more complicated theorems within this subject explored by the end of the project.