Subsequences

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Subsequence

Definition 2.6.1

The sequence $\{b_n\}_{n=i}^{\infty}$ is a *subsequence* of $\{a_n\}_{n=k}^{\infty}$, with $i, k \in \mathbb{N}, i \geq k$ if and only if there exists a strictly increasing function , where $f : \{m \in \mathbb{N} \mid m \geq i\} \to \{m \in \mathbb{N} \mid m \geq k\}$ and $b_n = a_{f(n)}$ for all $n \in \mathbb{N}$.

Subsequential Limit

Definition 2.6.3

α is a *subsequential limit point* of a sequence $\{a_n\}$ iff there exists a subsequence of $\{a_n\}$ that converges to α. Let T be the set of all subsequential limits of $\{a_n\}$. Then, $\sup T$ is called the *limit superior (upper limit)* of $\{a_n\}$, and we can write

$$\sup T = \limsup_{n\to\infty} a_n = \overline{\lim_{n\to\infty}}\, a_n$$

Similarly, $\inf T$ is called the *limit inferior (lower limit)* of $\{a_n\}$, and we can write

$$\inf T = \liminf_{n\to\infty} a_n = \underline{\lim_{n\to\infty}}\, a_n$$

Theorems on Subsequences

Theorem 2.6.4 (Bolzano-Weierstrass Theorem for Sequences)

Any bounded sequence must have at least one convergent subsequence.

Theorem 2.6.5

A sequence converges to A if and only if each of its subsequences converges to A.