Laboratory 6: Diffraction and Interference

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Physics 4BL Lab 8

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1 Introduction

In this lab, we will be performing experiments that demonstrate the wave-like nature of light. We will use laser beams, which are coherent and small in divergence to investigate single, double and multiple slit diffraction and interference. From the spectrum projected onto the screen, we can determine from maxima and minima, some parameters of the slit, including the slit width and slit separation. We will also conduct experiments on the diffraction of light by a piece of hair and a 2D grating. Equations used in this report are listed here:

$$bsin\theta = m\lambda \tag{1}$$

$$tan\theta = x/D \tag{2}$$

$$b\frac{x}{D} = m\lambda \tag{3}$$

$$d\frac{x}{D} = (m + \frac{1}{2})\lambda \tag{4}$$

$$d\frac{x}{D} = m\lambda \tag{5}$$

$$\sigma_F = \sqrt{(\frac{df}{dx})^2 \times (\sigma_x)^2 + (\frac{df}{dy})^2 \times (\sigma_y)^2}, \ given\ F = f(x, y) \tag{6}$$

Eqn.(1) is for the correlation between slit width (b), angular spacing (θ), minima number (m) and wavelength (λ) for single slit. Eqn.(2) is an approximation for the angular spacing. Eqn.(3) is resulted from plugging Eqn.(2) into Eqn.(1), in which x is the distance of minima from the central line and D is the distance from the slit to the screen. Eqn.(4) and Eqn.(5) are the equivalency of Eqn.(3) for a double slit, which are for minima and maxima respectively. Eqn.(6) is for error propagation.

2 Experimental Results

The instruments setup is shown in figure 1. For the diffraction grating part, we place an extra beam expander between the laser source and the gratings. The aperture is for narrowing the range of the maxima intensity.

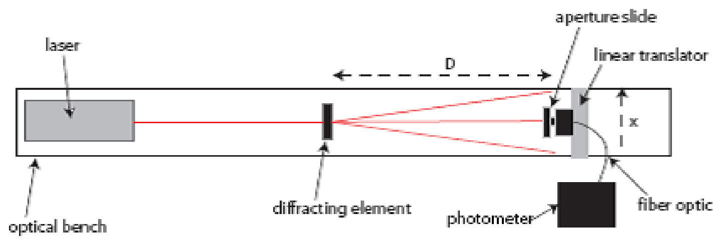

Figure 37: Schematic picture of the experimental setup and components.

Figure 1: Schematic picture of the experimental setup and components. The laser diode emits laser beam onto the slit or gratings, and the diffracting elements are either slits, gratings or human hair. The Linear translator is the rolling light detector, and the photometer receives the intensity of light falling on the detector.

We first test and record the detector response without any slits to obtain the position-voltage conversion mechanism. The laser beam profile is recorded is figure 2. We establish the conversion by assuming that the detector slides from $x = 1cm$ to $x = 4cm$ with a constant velocity (roughly $3sec/turn$) for all cases.

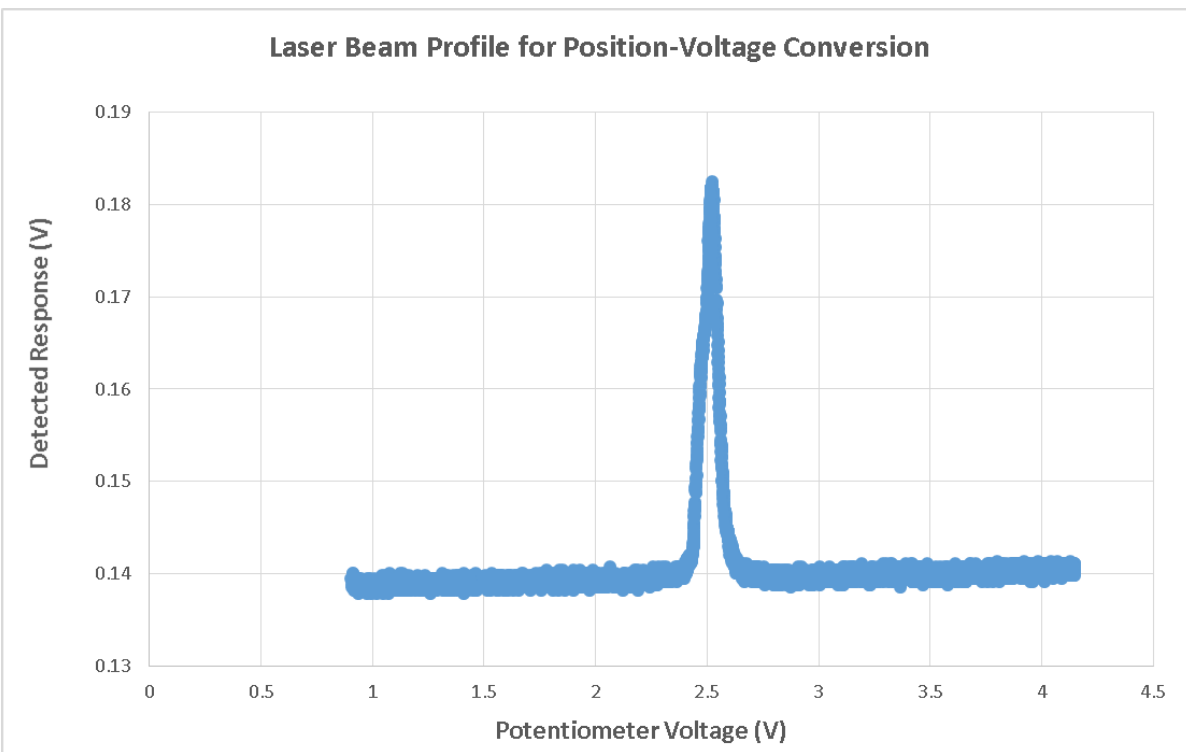

Figure 2: Graph of beam profile from a scan from $x = 1cm$ to $x = 4cm$. x axis is the voltage from the motion of the roller, and y axis is the voltage detected by the photometer. This profile will be used to convert voltage to position.

Since the amount of time used to sweep a distance of $(4-1)cm = 3cm = 0.03m$ is 90.81 seconds, the speed of the rolling motion of the detector is:

$$v = \frac{\Delta x}{\Delta t} = \frac{0.03m}{90.81s} = 0.00033m/s$$

And uncertainty is:

$$\sigma_v = \sqrt{(\frac{1}{t})^2 \times \sigma_x^2 + (-x \times t^{-2})^2 \times \sigma_t^2} = 0.00055m/s$$

So,

$$v = 0.0003 \pm 0.00055m/s = 0.3 \pm 0.6mm/s$$

2.1 Double Slits

In this section, we conduct measurements for a set of double slits.

First of all, we use a double slit with known parameters. After setting up the laser, the detector and the photometer, we placed between the laser and the detector a slide support and slipped into it a double slit with $b = 0.04mm$ and $d = 0.125mm$. With the laser on, we then roll the detector from $x = 1cm$ to $x = 4cm$ and record the voltage detected by the photometer with the myDAQ. The distance between the slit and the detector is 400mm, and the central maxima will occur at $x = 2.5cm$ since the single aperture is placed there so the photometer receives near full-scale voltage. Figure 3 is the diffraction pattern for the known double slit.

With the same experimental setup and methods, we replace the known double-slit with three unknown double slits and conduct spectrum measurements. These results are displayed in figure 4, 5, 6 respectively.

2.2 Diffraction Grating

For this part, we replace the double slits with a multi-slit grating consisting of 600lines/mm. We also use a beam expander to flatten the light onto the horizontal plane. We first use laser, then we use white light to shine on the grating to produce diffraction effect. Experimental setup is shown in figure 7.

The pattern observed by eye for the double slit is a sequence of lines in horizontal direction, while the pattern for the diffraction grating with laser light is a few really bright dots that align horizontally

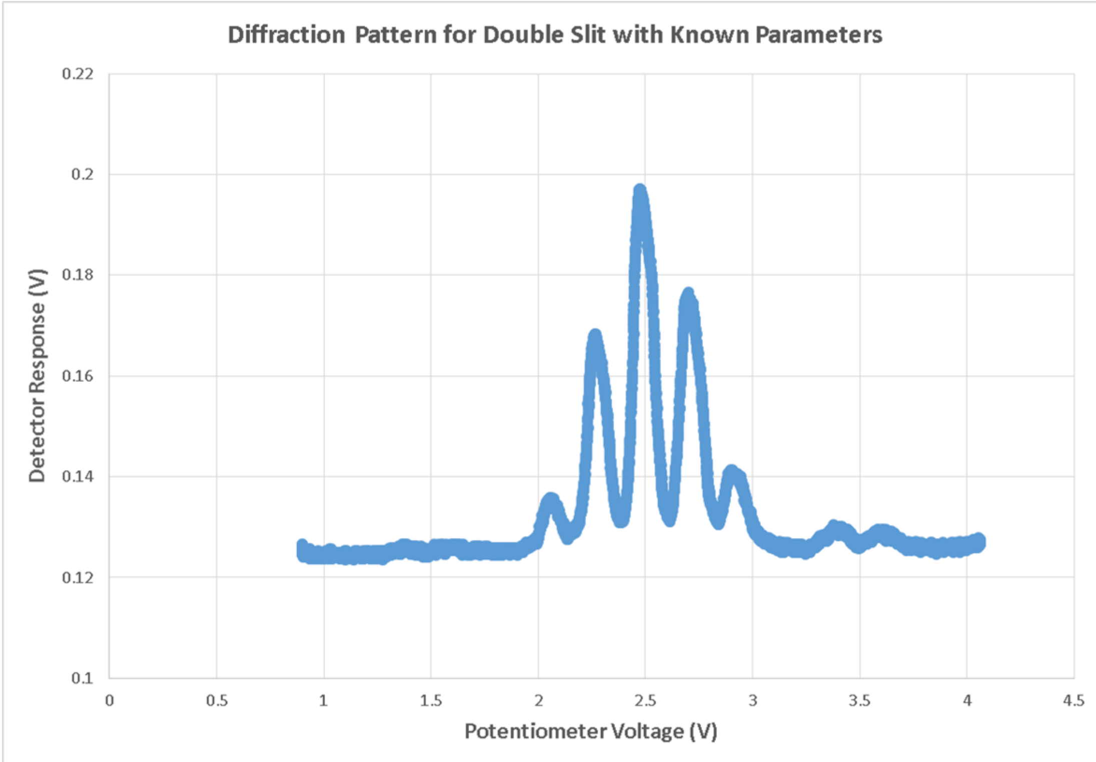

Figure 3: Graph of the diffraction pattern for the double slit with known parameters. The narrow maxima are due to double-slit effect and the overall envelope shape is due to the single-slit effect. We will verify the known parameters using data from this plot.

across the spectrum, and that for the white light shows decomposed light (red, green blue) beams align horizontally inside a few bright dots. For the diffraction grating, the maxima dots are farther apart and brighter.

Measured angular spacing for laser light is recorded in table 1, and that for white light is recorded in table 2. Angles are read off symmetrically from both sides of the spectrum, and will be used to verify Eqn.(5).

Diffraction Grating with Laser			
m	-1	0	1
Angle (deg)	65.0	90.0	114.5

Table 1: Raw data for the diffraction grating with laser light. The light is flattened onto a sheet by a beam expander. We measure the diffraction spectrum with a protractor and record the angle for maxima at m = -1, 0 and 1.

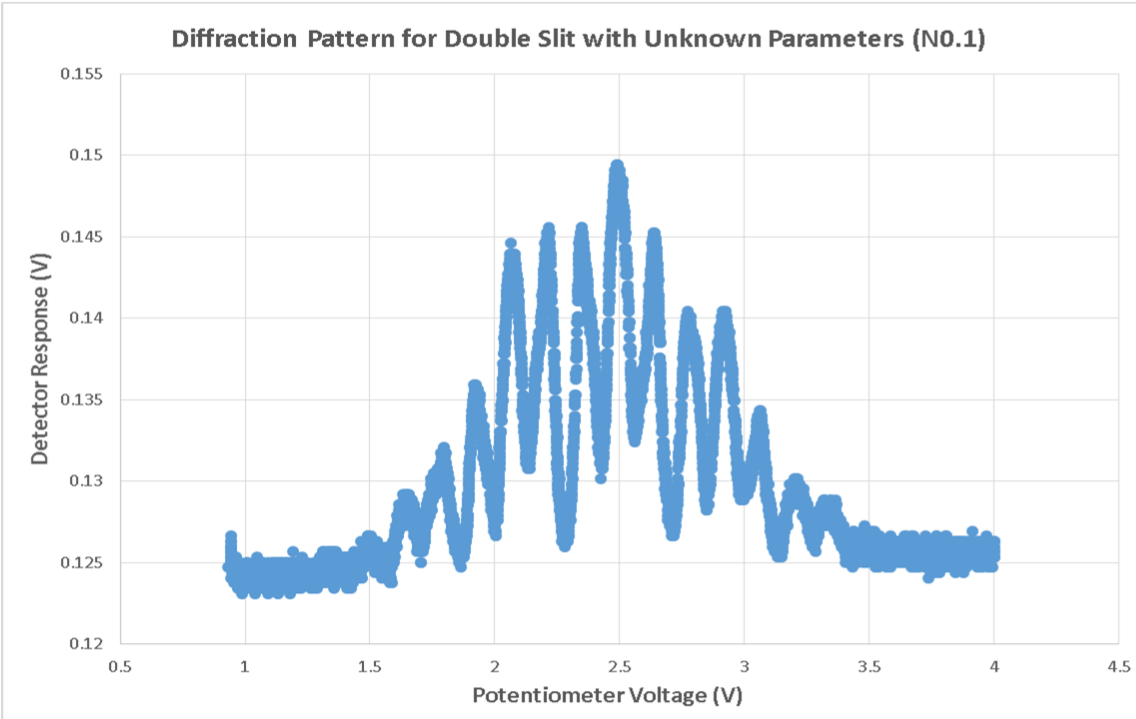

Figure 4: Graph of the diffraction pattern for the first double slit with unknown parameters. We will compute the parameters (b, d) using data from this plot.

2.3 Additional Experiments

We choose to perform the human hair experiment. The observed diffraction pattern, the same as the double-slit one, gives us a sequence of horizontal lines on a white paper. At m = 1, $\delta x = 6.0mm$, which is the distance between the fist maxima and the central maxima.

3 Analysis

3.1 Double slit with known parameters

In this section we will verify the given slit width and spacing of a double slit with known parameters. Refer to figure 3 and the rolling speed v computed in the previous section for this part.

Choosing m = -1 and m = 1 (i.e. the first maxima to the left and to the right of the central maxima) on figure 3, we obtain, for m = -1:

$$x_{-1} = 0.025m - (\Delta t \times v + 0.01m) = 0.00227m \tag{7}$$

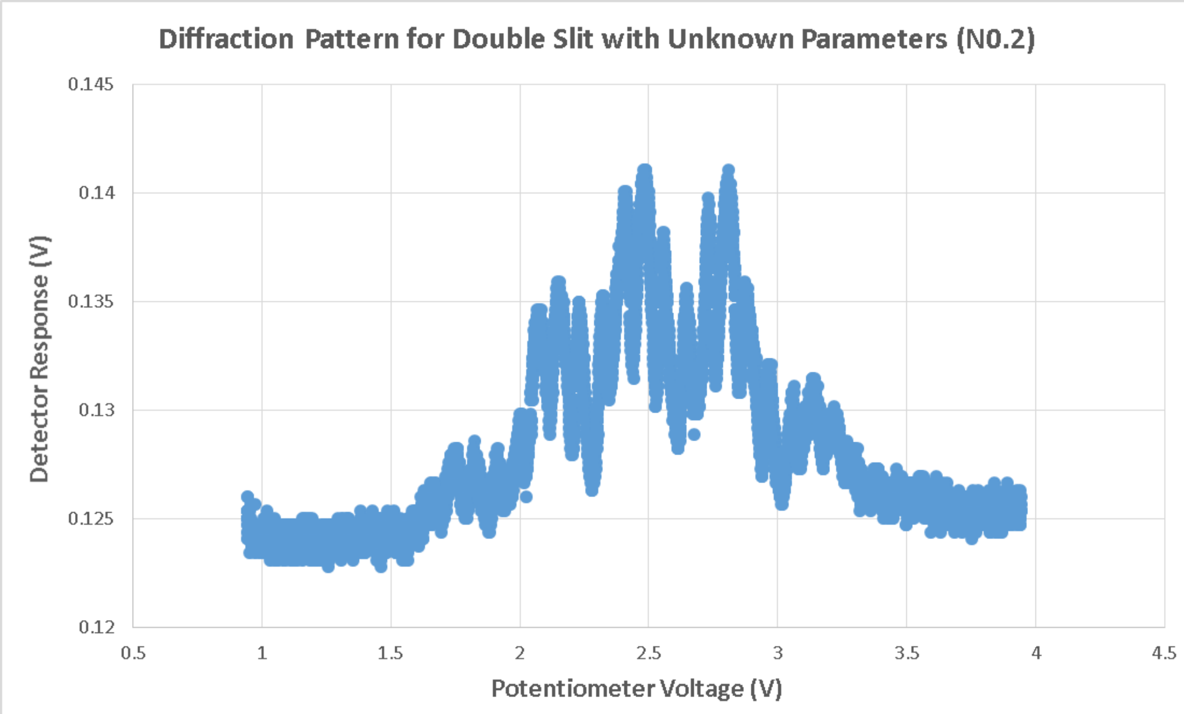

Figure 5: Graph of the diffraction pattern for the second double slit with unknown parameters. We will compute the parameters (b, d) using data from this plot.

And for m = 1:

$$x_1 = (\Delta t \times v + 0.01m) - 0.025m = 0.00151m$$

Where Δt is read off from the time-variable column in our excel spreadsheet, and $\Delta t \times v$ gives the distance rolled over by the detector from x = 1cm.

The uncertainty in x is given by:

$$\sigma_x = \sqrt{(\frac{df}{dt})^2 \times (\sigma_t)^2 + (\frac{df}{dv})^2 \times (\sigma_v)^2} = 0.004mm$$

Plugging x into Eqn.(5), and with some arrangements, we obtain the slit spacing:

$$d_{-1} = \frac{m\lambda}{x/D} = \frac{670E-09}{0.00227/0.4} = 0.118mm \tag{8}$$

Using Eqn.(6), the uncertainty is:

$$\sigma_{d_{-1}} = \lambda \times \sqrt{(\frac{df}{dx})^2 \times (\sigma_x)^2 + (\frac{df}{dD})^2 \times (\sigma_D)^2} = 0.009mm$$

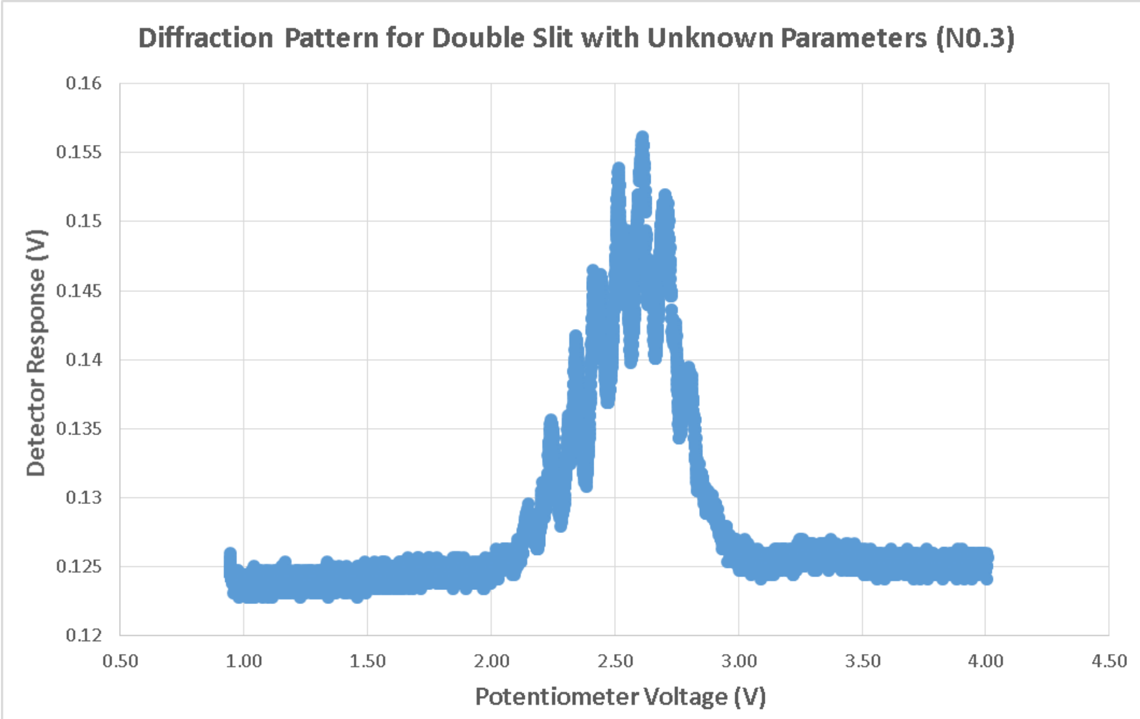

Figure 6: Graph of the diffraction pattern for the third double slit with unknown parameters. We will compute the parameters (b, d) using data from this plot.

So for m = -1,

$$d_{-1} = 0.118 \pm 0.009mm$$

And the provided value d=0.125mm is within this uncertainty range.

Similarly, For m = 1,

$$d_1 = \frac{m\lambda}{x/D} = \frac{670E-09}{0.00151/0.4} = 0.177mm$$

And uncertainty is;

$$\sigma_{d_{-1}} = \lambda \times \sqrt{(\frac{df}{dx})^2 \times (\sigma_x x)^2 + (\frac{df}{dD})^2 \times (\sigma_D)^2} = 0.06mm$$

So for m = 1,

$$d_1 = 0.18 \pm 0.06mm$$

And the provided value d=0.125mm also falls within this uncertainty range. So both cases are able to verify the given d value to be true.

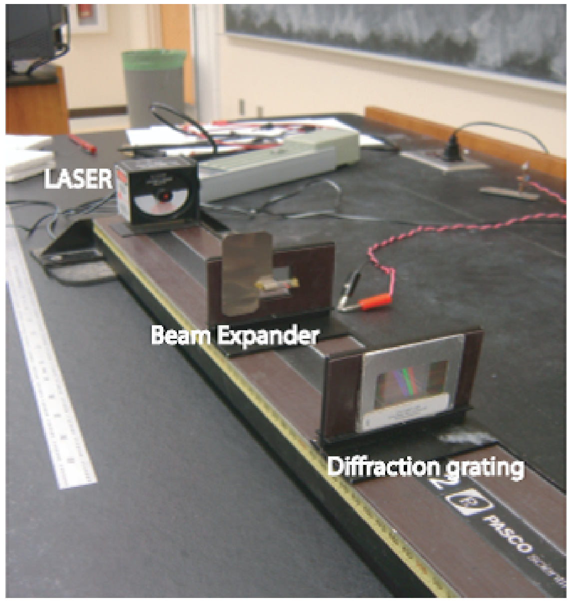

Figure 7: Laser, beam expander and diffraction grating. The beam expander is used to produce a sheet beam of light. Photometer will be replaced by a protractor to measure the angular spacing.

Diffraction Grating with White Light			
Parameters	Red	green	blue
left	75.0	70.0	68.0
central	90.0	90.0	90.0
right	105.0	110.0	112.0

Table 2: Raw data for the diffraction grating with white light. The light is flattened onto a sheet by a beam expander and are separated into different colors due to difference in wavelength. We measure the diffraction spectrum with a protractor and record the angle for maxima at central maxima, to its left, and to its right.

Since the overall envelope shape is not obvious for this part, we will not compute the slit width for it.

3.2 Double slit with unknown parameters

Double slit No.1

For the first double slit, refer to figure 4. Since figure 4 doesn't show distinguishable overall envelope shape, we will only compute the slit spacing but not the slit width. To minimize the error in our calculation, we will pick eight maxima (four to the left of the central maxima, and four to the right), obtain a slit spacing from each maxima, and compute the average. The uncertainty will be calculated from statistical method. Values from the eight maxima are shown in table 3.

Double Slit With Unknown Parameters No.1		
m	x (m)	d (mm)
1	0.00108	0.248
2	0.00224	0.40
3	0.00355	0.226
4	0.00487	0.220
-1	0.00163	0.165
-2	0.00300	0.179
-3	0.00452	0.178
-4	0.00587	0.183

Table 3: Showing the location of maxima (x) and the correspondingly calculated slit spacing (d) from each maxima. The maxima are chosen and read off from figure 4 for double slit No.1. m is the maxima number $0, \pm1, \pm2....$

The x values in table 3 are calculated in the same way as Eqn.(7) in section 3.1, which is (if $m < 0$):

$$x = 0.025m - (\Delta t \times v + 0.01m)$$

Or (if $m > 0$),

$$x = (\Delta t \times v + 0.01m) - 0.025m$$

and d values in table 3 are calculated in the same way as Eqn.(8) in section 3.1, which is:

$$d = \frac{m\lambda}{x/D} = \frac{m \times 670E - 09}{x/0.4}$$

The average of d in table 3 is:

$$d_{ave} = 0.2048mm$$

And the standard deviation of d is:

$$d_{stdev} = 0.0322mm$$

Based on the uncertainty formula, the uncertainty of d is:

$$\sigma_d = \frac{d_{stdev}}{\sqrt{N}} = 0.0114mm$$

So,

$$d = 0.20 \pm 0.01mm$$

Double slit No.2

This part and the next part on the double slits are computationally the same as the previous part. We will show the data we calculated from selected maxima just like we did before. For the second double slit, refer to figure 5. Table 4 record x and d values obtained from six maxima on figure 5 (three to the left of the central maxima, and three to the right).

Double Slit With Unknown Parameters No.2 – d		
m	x (m)	d (mm)
2	0.00110	0.486
3	0.00203	0.396
4	0.00292	0.367
-1	0.00124	0.216
-2	0.00202	0.266
-3	0.00293	0.274

Table 4: Showing the location of maxima (x) and the correspondingly calculated slit spacing (d) from each maxima. The maxima are chosen and read off from figure 5 for double slit No.2. m is the maxima number $0, \pm 1, \pm 2....$

The average of d in table 4 is:

$$d_{ave} = 0.334mm$$

And the standard deviation of d is:

$$d_{stdev} = 0.1mm$$

Based on the uncertainty formula, the uncertainty of d is:

$$\sigma_d = \frac{d_{stdev}}{\sqrt{N}} = 0.041mm$$

So,

$$d = 0.33 \pm 0.04mm$$

Since the second double slit has the largest slit spacing, and in figure 5 we can recognize an overall envelope shape, we will calculate the slit width using it. Instead of adjacent maxima, for the slit width, we will pick minima. Values calculated from minima in figure 5 are included in table 5.

Double Slit With Unknown Parameters No.2 – b		
m	x (m)	b (mm)
1	0.00082	0.327
2	0.00551	0.0973
-1	0.00241	0.111
-2	0.00618	0.0867

Table 5: Showing the location of minima (x) and the correspondingly calculated slit width (b) from each minima. The minima are chosen and read off from figure 5 for double slit No.2. m is the minima number $\pm 1, \pm 2, \pm 3$....

The x values in table 5 are calculated in the same way as Eqn.(7) in section 3.1, which is (if $m < 0$):

$$x = 0.025m - (\Delta t \times v + 0.01m)$$

Or (if $m > 0$),

$$x = (\Delta t \times v + 0.01m) - 0.025m$$

and b values in table 5 are calculated in Eqn.(3), which is:

$$b = \frac{m\lambda}{x/D} = \frac{m \times 670E - 09}{x/0.4}$$

The average of b in table 5 is:

$$b_{ave} = 0.156mm$$

And the standard deviation of d is:

$$b_{stdev} = 0.115mm$$

Based on the uncertainty formula, the uncertainty of d is:

$$\sigma_b = \frac{b_{stdev}}{\sqrt{N}} = 0.057mm$$

So,

$$b = 0.16 \pm 0.06mm$$

Double slit No.3

For the third and last double slit, refer to figure 6. Table 6 record x and d values obtained from six maxima on figure 6 (three to the left of the central maxima, and three to the right).

Double Slit With Unknown Parameters No.3 – d		
m	x (m)	d (mm)
1	0.00078	0.342
2	0.00159	0.338
3	0.00244	0.329
-1	0.000949	0.283
-2	0.00163	0.329
-3	0.00251	0.320

Table 6: Showing the location of maxima (x) and the correspondingly calculated slit spacing (d) from each maxima. The maxima are chosen and read off from figure 6 for double slit No.3. m is the maxima number $0, \pm 1, \pm 2....$

The average of d in table 6 is:

$$d_{ave} = 0.324mm$$

And the standard deviation of d is:

$$d_{stdev} = 0.0215mm$$

Based on the uncertainty formula, the uncertainty of d is:

$$\sigma_d = \frac{d_{stdev}}{\sqrt{N}} = 0.0088mm$$

So,

$$d = 0.324 \pm 0.009mm$$

Summary of section 3.2—-double slits with unknown parameters

Based on data in table 3, 4, 6, we can see that as the slit spacing increases, the separation between maxima in the spectrum decreases.

3.3 Diffraction Grating

Laser Light

For this part, refer to table 1.

Based on Eqn.(1),

$$d = \frac{m\lambda}{sin\theta}$$

Where θ is the angular spacing for maxima. From table 1, we obtain,

$$d_{-1} = 0.00067/sin(25) = 0.00159mm$$

The uncertainty in reading the protractor is 0.5 deg. Converting this to radian, we get:

$$\sigma_{\theta_{rad}} = \frac{\pi}{180} \times \sigma_{\theta_{deg}} = \frac{\pi}{180} \times 0.5 = 0.009rad$$

Then according to the error propagation equation, the uncertainty in d is:

$$\sigma_d = \lambda \times (-sin\theta^{-2} cos\theta \times \sigma_\theta)$$

So

$$\sigma_d = 3E - 05mm$$

So,

$$d = 0.00159 \pm 0.00003mm$$

The provided value of d is:

$$d_{provided} = 1/559 = 0.00167mm$$

Which falls outside the uncertainty range of our calculated value. This could be due to mishandling of the protractor readings.

White light

This part uses the measured grating spacing from the previous part. Refer to table 2.

For the red light,

$$frequency = \frac{C}{d \times sin\theta} = 9.08E + 14(Hz)$$

And uncertainty is:

$$\sigma_f = C \times \sqrt{(\frac{df}{dd})^2 \times (\sigma_d)^2 + (\frac{df}{d\theta})^2 \times (\sigma_\theta)^2} = 0.000007E + 14(Hz)$$

So,

$$f_r = (9.080000 \pm 0.000007)E + 14(Hz)$$

Similarly, for the blue light,

$$f_b = (1.270000 \pm 0.000004)E + 15(Hz)$$

So the bandwidth of the visible light is:

$$f_b - f_r = (3.620000 \pm 0.000003)E + 14(Hz)$$

3.4 Addition Experiment—-human hair

For this last part of the lab, we measure the thickness of human hair using diffraction.

The thickness is given by:

$$d = \frac{\lambda}{x/D} = 0.0949mm$$

Uncertainty is:

$$\sigma_d = \lambda \times \sqrt{(\frac{df}{dx})^2 \times (\sigma_x)^2 + (\frac{df}{dD})^2 \times (\sigma_D)^2} = 0.003mm$$

So,

$$d = 0.095 \pm 0.003mm$$

4 Conclusion

In this lab, we succeeded in verifying the wave-like nature of light by doing double slits, diffraction gratings and human hair. We verified the double slit with known parameters, and we measured double slits with unknown parameters. We also Verified the spacing of a diffraction grating and measured the bandwidth of visible light. In the end, we measured the thickness of human hair. We also learned how to use the photometer and the linear translator to stimulate the diffraction and interference of waves, and we strengthened our familiarity with error propagation. Thank you Eddie!!